Verification of Security Protocols

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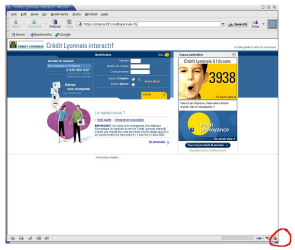
Context : cryptographic protocols

Cryptographic protocols are widely used in everyday life.

Credit card payment

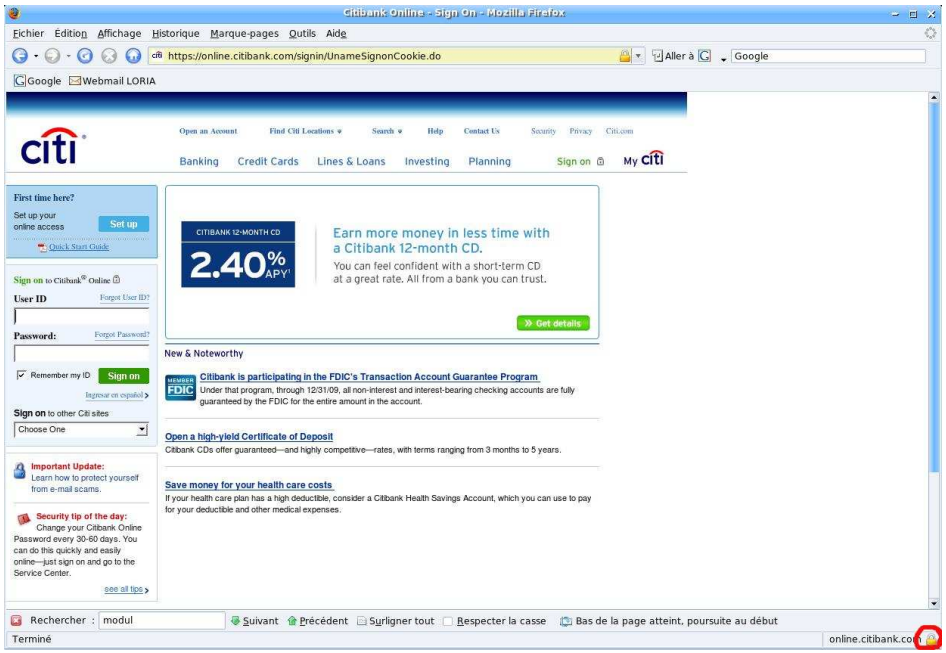
On the web (SSH, SSL, ...)



Pay-per-view

Electronic purse

On the web



- HTTPS, i.e. the SSL protocol for ensuring confidentiality

- password-based authentication

Credit Card payment

- It is a real card ?
- Is the pin code protected ?

Example : Credit Card Payment Protocol

- The waiter introduces the credit card.
- The waiter enters the amount m of the transaction on the terminal.
- The terminal authenticates the card.
- The customer enters his secret code.
  If the amount m is greater than 100 euros
  (and in only 20% of the cases)
  - The terminal asks the bank for authentication of the card.
  - The bank provides authentication.

More details

4 actors : Bank, Customer, Card and Terminal.

Bank owns

- a signing key K_B^{-1}, secret,
- a verification key K_B, public,
- a secret symmetric key for each credit card K_{CB}, secret.

Card owns

- Data : last name, first name, card's number, expiration date,
- Signature's Value $VS = \{hash(\text{Data})\}_{K_B^{-1}}$,
- secret key K_{CB}.

Terminal owns the verification key K_B for bank's signatures.

Credit card payment Protocol (in short)

The terminal reads the card :

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$$3. \quad Cu \rightarrow Ca : 1234$$
$$4. \quad Ca \rightarrow T : ok$$

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The terminal calls the bank :

$$5. \quad T \rightarrow B : auth?$$
$$6. \quad B \rightarrow T : N_b$$
$$7. \quad T \rightarrow Ca : N_b$$
$$8. \quad Ca \rightarrow T : \{N_b\}_{K_{CB}}$$
$$9. \quad T \rightarrow B : \{N_b\}_{K_{CB}}$$
$$10. \quad B \rightarrow T : ok$$

Some flaws

The security was initially ensured by :

- the cards were very difficult to reproduce,
- the protocol and the keys were secret.

But

- cryptographic flaw : 320 bits keys can be broken (1988),
- logical flaw : no link between the secret code and the authentication of the card,
- fake cards can be build.

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→ "YesCard" build by Serge Humpich
(1998 in France).

How does the "YesCard" work ?

Logical flaw

1. $Ca \rightarrow T$: Data, $\{hash(\text{Data})\}_{K_B^{-1}}$
2. $T \rightarrow Ca$: *secret code?*
3. $Cu \rightarrow Ca$: 1234
4. $Ca \rightarrow T$: *ok*

How does the "YesCard" work ?

Logical flaw

1. $Ca \longrightarrow T$: Data, $\{hash(\text{Data})\}_{K_B^{-1}}$
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3. $Cu \longrightarrow Ca'$: 2345
4. $Ca' \longrightarrow T$: *ok*

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$\rightarrow$ creation of a fake card

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1. $Ca' \rightarrow T$: XXX, $\{hash(\text{XXX})\}_{K_B^{-1}}$
2. $T \rightarrow Cu$: *secret code?*
3. $Cu \rightarrow Ca'$: 0000
4. $Ca' \rightarrow T$: *ok*

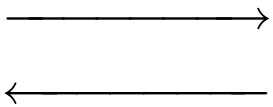
Electronic signature

- authenticates the signer

- should be verifiable by anyone

- ensures non-repudiation
(I never signed that message !)

Pay-per-view devices



- Checks your identity
- You should be granted access to the movie only once
- You should not be able to broadcast the movie to other people

Electronic voting

- The result corresponds to the votes.
- Each vote is confidential.
- No partial result is leaked before the end of the election
- Only voters can vote and at most once
- Coercion resistance

Electronic purse

- It should not possible to add money without paying.
- It should not be possible to create fake electronic purse.

Secure databases

Patients	Social security number	Diseases	Treatments
Smith	123 - 45 - 6789	Flu	...
Smith	123 - 45 - 6789	Arthritis	...
Williams	789 - 56 - 1234	Dehydration	...
Johnson	012 - 34 - 5678	Arthritis	...

- Authorized persons have access only to a partial view of the database (different for doctors, nurses, researchers, ...)
- Data may be exchanged between e.g. doctors, hospitals, medical laboratories.
- Data are regularly updated.

Security goals

Cryptographic protocols aim at

- preserving confidentiality of data
  (e.g. pin code, medical files, ...)
- ensuring authenticity
  (Are you really talking to your bank ? ?)
- ensuring anonymous communications
  (for e-voting protocols, ...)
- protecting against repudiation
  (I never sent this message ! !)
- ...

$\Rightarrow$ Cryptographic protocols vary depending on the application.

Difficulty

Presence of an attacker

- may read every message sent on the net,
- may intercept and send new messages.

$\Rightarrow$ The system is infinitely branching

Outline of the talk

Motivation : Cryptography does not suffice to ensure security !

Example : Commutative encryption (RSA)

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$$\text{Since } \left\{\{\text{pin} : 3443\}_{k_{\text{alice}}}\right\}_{k_{\text{bob}}} = \left\{\{\text{pin} : 3443\}_{k_{\text{bob}}}\right\}_{k_{\text{alice}}}$$

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$\longrightarrow$ It does not work ! (Authentication problem)

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→ It does not work ! (Authentication problem)

$$\{\text{pin} : 3443\}_{k_{\text{alice}}} \longrightarrow$$

$$\leftarrow \left\{\{\text{pin} : 3443\}_{k_{\text{alice}}}\right\}_{k_{\text{intruder}}}$$

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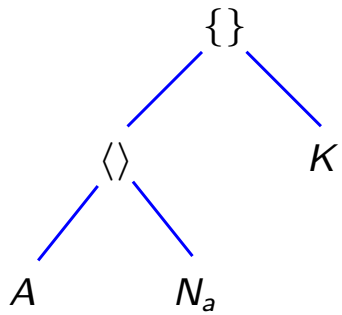
Messages

Messages are abstracted by terms.

Agents : $a, b, \ldots$ Nonces : $n_1, n_2, \ldots$ Keys : $k_1, k_2, \ldots$
Cyphertext : $\{m\}_k$ Concatenation : $\langle m_1, m_2 \rangle$

Example : The message $\{A, N_a\}_K$ is represented by :



Intruder abilities

Composition rules

$$\frac{T \vdash u \quad T \vdash v}{T \vdash \langle u\,, v\rangle} \qquad \frac{T \vdash u \quad T \vdash v}{T \vdash \mathsf{enc}(u, v)} \qquad \frac{T \vdash u \quad T \vdash v}{T \vdash \mathsf{enca}(u, v)}$$

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$$\frac{}{T \vdash u}\, u \in T \qquad \frac{T \vdash \langle u\,, v\rangle}{T \vdash u} \qquad \frac{T \vdash \langle u\,, v\rangle}{T \vdash v}$$

$$\frac{T \vdash \mathrm{enc}(u, v) \quad T \vdash v}{T \vdash u} \qquad \frac{T \vdash \mathrm{enca}(u, \mathrm{pub}(v)) \quad T \vdash \mathrm{priv}(v)}{T \vdash u}$$

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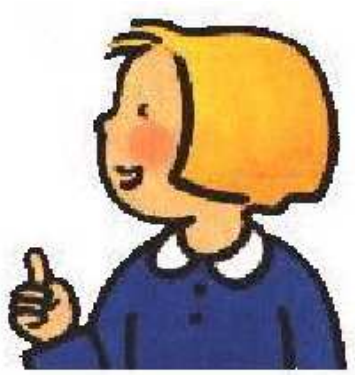
Deducibility relation

A term u is deducible from a set of terms T, denoted by $T \vdash u$, if there exists a prooftree witnessing this fact.

A simple protocol

$\langle Bob, k \rangle$ →

← $\langle Alice, enc(s, k) \rangle$



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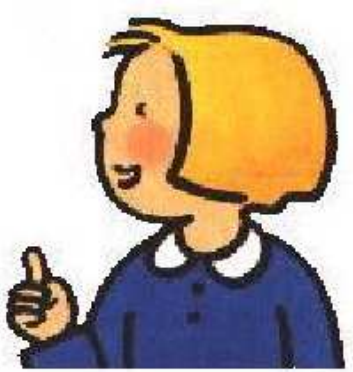

Question ?

Can the attacker learn the secret s ?

A simple protocol

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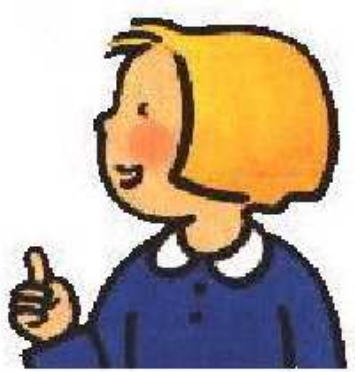

Answer : Of course, Yes !

$$\frac{\dfrac{\langle Alice, enc(s, k) \rangle}{enc(s, k)} \qquad \dfrac{\langle Bob, k \rangle}{k}}{s}$$

Decision of the intruder problem

Given A set of messages S and a message m

Question Can the intruder learn m from S that is $S \vdash m$?

This problem is decidable in polynomial time

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Lemma (Locality)

If there is a proof of $S \vdash m$ then there is a proof that only uses the subterms of S and m.

Protocol description

Protocol :

$$\begin{array}{lcl} A \longrightarrow B & : & \{\mathsf{pin}\}_{k_a} \\ B \longrightarrow A & : & \{\{\mathsf{pin}\}_{k_a}\}_{k_b} \\ A \longrightarrow B & : & \{\mathsf{pin}\}_{k_b} \end{array}$$

A protocol is a finite set of roles :

- role $\Pi(1)$ corresponding to the 1^{st} participant played by a talking to b :

$$\begin{array}{rcl} \mathsf{init} & \xrightarrow{k_a} & \mathsf{enc}(\mathsf{pin}, k_a) \\ \mathsf{enc}(x, k_a) & \longrightarrow & x. \end{array}$$

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- role $\Pi(2)$ corresponding to the 2^{nd} participant played by b with a :

$$\begin{array}{rcl} x & \xrightarrow{k_b} & \mathsf{enc}(x, k_b) \\ \mathsf{enc}(y, k_b) & \to & \mathsf{stop}. \end{array}$$

Secrecy via constraint solving [Millen et al]

Constraint systems are used to specify secrecy preservation under a particular, finite scenario.

Scenario

$$\begin{array}{l} \mathrm{rcv}(u_1) \xrightarrow{N_1} \mathrm{snd}(v_1) \\ \mathrm{rcv}(u_2) \xrightarrow{N_2} \mathrm{snd}(v_2) \\ \quad \ldots \\ \mathrm{rcv}(u_n) \xrightarrow{N_n} \mathrm{snd}(v_n) \end{array}$$

Constraint System

$$\mathcal{C} = \begin{cases} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \ldots \\ T_0, v_1, .., v_n \Vdash s \end{cases}$$

where T_0 is the initial knowledge of the attacker.

Remark : Constraint Systems may be used more generally for trace-based properties, e.g. authentication.

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where T_0 is the initial knowledge of the attacker.

Solution of a constraint system

A substitution σ such that

for every $T \Vdash u \in \mathcal{C}$, $u\sigma$ is deducible from $T\sigma$, that is $u\sigma \vdash T\sigma$.

Example of a system constraint

$$\begin{array}{lcl} A \to B & : & \{\mathsf{pin}\}_{k_a} \\ B \to A & : & \{\{\mathsf{pin}\}_{k_a}\}_{k_b} \\ A \to B & : & \{\mathsf{pin}\}_{k_b} \end{array}$$ and the attacker initially knows $T_0 = \{\mathsf{init}\}$.

One possible associated constraint system is :

$$\mathcal{C} = \left\{ \begin{array}{l} \{\mathsf{init}\} \Vdash \mathsf{init} \\ \{\mathsf{init}, \{\mathsf{pin}\}_{k_a}\} \Vdash \{\mathsf{x}\}_{k_a} \\ \{\mathsf{init}, \{\mathsf{pin}\}_{k_a}, x\} \Vdash \mathsf{pin} \end{array} \right.$$

Is there a solution ?

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Is there a solution ?

Of course yes, simply consider $x = \mathsf{pin}$!

How to solve constraint system ?

$$\text{Given } \mathcal{C} = \begin{cases} T_0 \Vdash u_1 \\ T_0, v_1 \Vdash u_2 \\ \quad \dots \\ T_0, v_1, .., v_n \Vdash u_{n+1} \end{cases}$$

Question Is there a solution σ of $\mathcal{C}$?

An easy case : "solved constraint systems"

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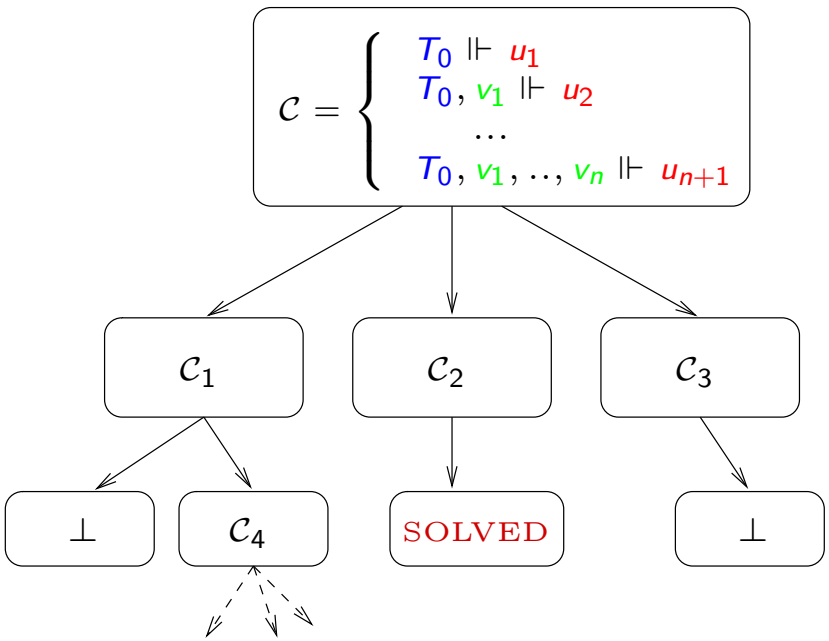
Of course yes !

Consider e.g. $\sigma(x_1) = \cdots = \sigma(x_{n+1}) = t \in T_0$.

Decision procedure [Millen / Comon-Lundh]

Goal : Transformation of the constraints in order to obtain a solved constraint system.



$\mathcal{C}$ has a solution iff $\mathcal{C} \rightsquigarrow \mathcal{C}'$ with $\mathcal{C}'$ in solved form.

Intruder step

The intruder can built messages

$$R_5 : \quad \mathcal{C} \wedge T \Vdash f(u,v) \quad \rightsquigarrow \quad \mathcal{C} \wedge T \Vdash u \wedge T \Vdash v \qquad \text{for } f \in \{\langle\rangle, \mathrm{enc}, \mathrm{enca}\}$$

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Example :

$$a, k \Vdash \text{enc}(\langle x, y\rangle, k) \quad \rightsquigarrow \quad \begin{array}{l} a, k \Vdash k \\ a, k \Vdash x \\ a, k \Vdash y \end{array}$$

Eliminating redundancies

$$k \Vdash x$$
$$k, \mathrm{enc}(s, x) \Vdash s$$

The constraint $\mathrm{enc}(s, x) \Vdash s$ will be satisfied as soon as $k \Vdash x$ is satisfied.

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$$R_1 : \mathcal{C} \wedge T \Vdash u \rightsquigarrow \mathcal{C} \quad \text{if } T \cup \{x \mid T' \Vdash x \in \mathcal{C}, T' \subsetneq T\} \vdash u$$

Unsolvable constraints

$$R_4 : \mathcal{C} \wedge T \Vdash u \rightsquigarrow \bot \qquad \text{if } \mathrm{var}(T, u) = \emptyset \text{ and } T \not\vdash u$$

Example :

$$\begin{array}{l} \ldots \\ a, \mathrm{enc}(s, k) \Vdash s \quad \rightsquigarrow \quad \bot \\ \ldots \end{array}$$

Guessing equalities

1. Example : $k, \mathrm{enc}(\mathrm{enc}(x, k'), k) \Vdash \mathrm{enc}(a, k')$

$$R_2 : \mathcal{C} \wedge T \Vdash u \rightsquigarrow_\sigma \mathcal{C}\sigma \wedge T\sigma \Vdash u\sigma \qquad u' \in st(T)$$
$$\text{if } \sigma = \mathrm{mgu}(u, u'),\ u, u' \notin \mathcal{X},\ u \neq u'$$

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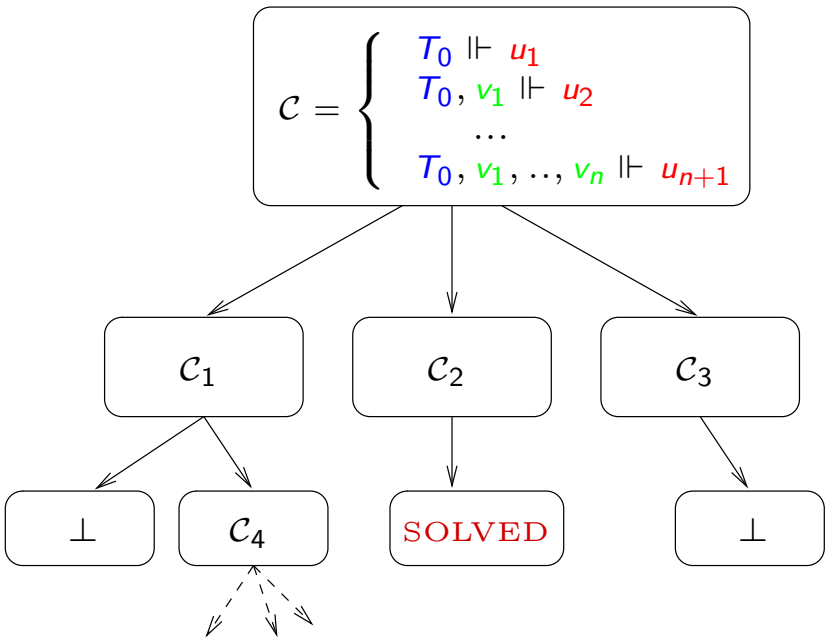
2. Example : $\mathrm{enc}(s, \langle a, x \rangle), \mathrm{enc}(\langle y, b \rangle, k), k \Vdash s$

$$R_3 : \mathcal{C} \wedge T \Vdash v \rightsquigarrow_\sigma \mathcal{C}\sigma \wedge T\sigma \Vdash v\sigma \qquad u, u' \in st(T)$$
$$\text{if } \sigma = \mathrm{mgu}(u, u'),\ u, u' \notin \mathcal{X},\ u \neq u'$$

NP-procedure for solving constraint systems



Theorem

- *$\mathcal{C}$ has a solution iff $\mathcal{C} \rightsquigarrow \mathcal{C}'$ with $\mathcal{C}'$ in solved form.*
- *$\rightsquigarrow$ is terminating in polynomial time.*

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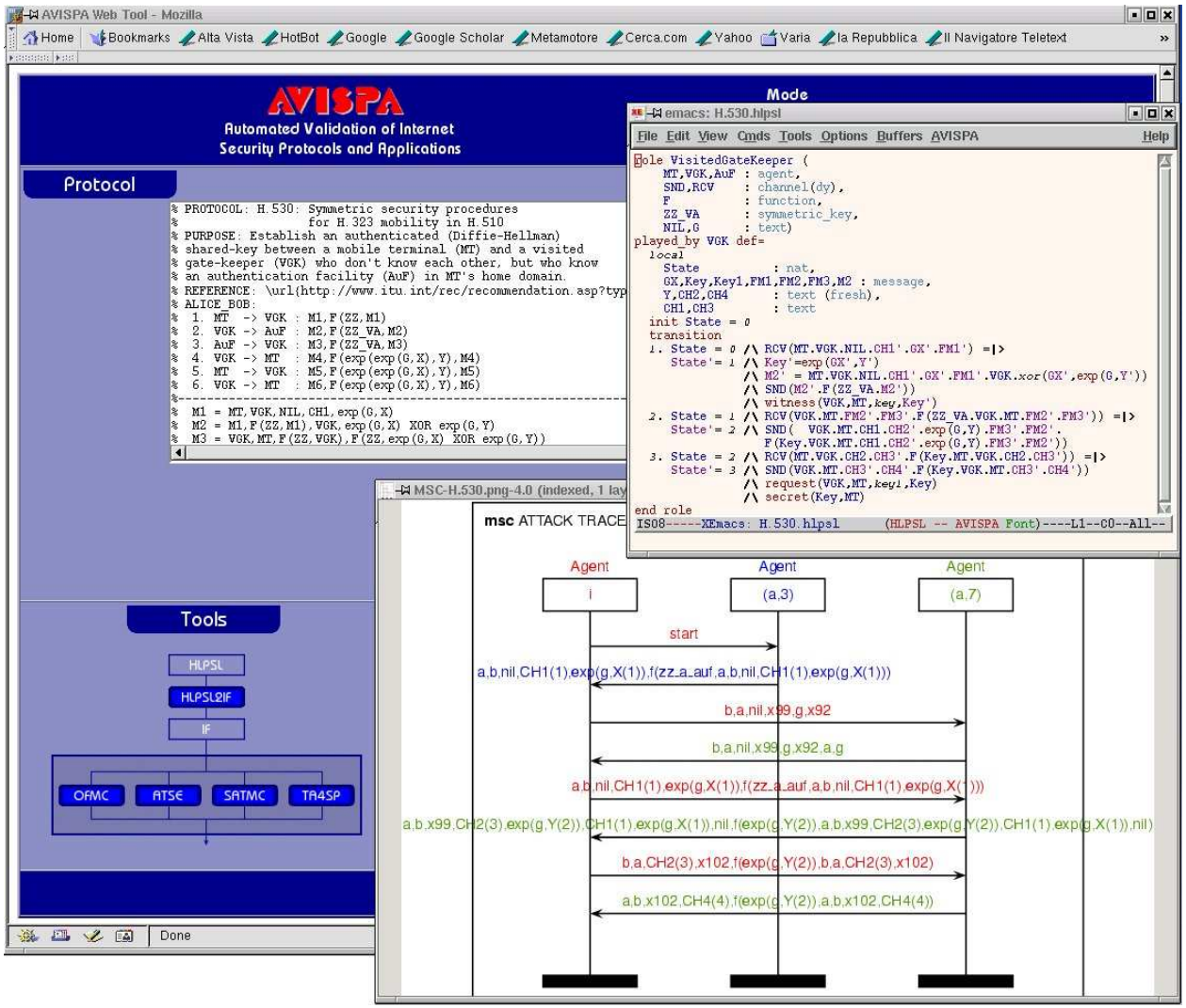
- For an unbounded number of sessions
  - for one-copy protocols, secrecy is DEXPTIME-complete [CortierComon RTA03] [SeildVerma LPAR04]
  - for message-length bounded protocols, secrecy is DEXPTIME-complete [Durgin et al FMSP99] [Chevalier et al CSL03]

  $\longrightarrow$ some tools for proving security (ProVerif, EVA Platform)

Example of tool : Avispa Platform



Collaborators

- LORIA, France
- DIST, Italy
- ETHZ, Switzerland
- Siemens, Germany

www.avispa-project.org

Outline of the talk

Motivation

Back to our running example :

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We need the equation for the commutativity of encryption

$$\{\{z\}_x\}_y = \{\{z\}_y\}_x$$

Some other examples

Encryption-Decryption theory

$$\mathsf{dec}(\mathsf{enc}(x,y),y) = x \quad \pi_1(\langle x,y\rangle) = x \quad \pi_2(\langle x,y\rangle) = y$$

EXclusive Or

$$\begin{array}{rcl@{\qquad}rcl} x \oplus (y \oplus z) & = & z & x \oplus y & = & y \oplus x \\ x \oplus x & = & 0 & x \oplus 0 & = & x \end{array}$$

Diffie-Hellmann

$$\exp(\exp(z,x),y) = \exp(\exp(z,y),x)$$

E-voting protocols

First phase :

$$V \to A : \quad \mathrm{sign}(\mathit{blind}(\mathrm{vote}, r), V)$$
$$A \to V : \quad \mathrm{sign}(\mathit{blind}(\mathrm{vote}, r), A)$$

Voting phase :

$$V \to C : \quad \mathrm{sign}(\mathrm{vote}, A)$$

...

Equational theory for blind signatures

[Kremer Ryan 05]

$$\begin{aligned} \mathsf{checksign}(\mathsf{sign}(x,y),\mathsf{pk}(y)) &= x \\ \mathsf{unblind}(\mathsf{blind}(x,y),y) &= x \\ \mathsf{unblind}(\mathsf{sign}(\mathsf{blind}(x,y),z),y) &= \mathsf{sign}(x,z) \end{aligned}$$

Deduction

$$\frac{}{T \vdash_E M} M \in T \qquad \frac{T \vdash_E M_1 \quad \cdots \quad T \vdash_E M_k}{T \vdash_E f(M_1, \ldots, M_k)} f \in \Sigma$$

$$\frac{T \vdash M}{T \vdash M'} M =_E M'$$

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Example : $E := \mathrm{dec}(\mathrm{enc}(x, y), y) = x$ and $T = \{\mathrm{enc}(\mathit{secret}, k),\ k\}$.

$$\dfrac{\dfrac{\dfrac{}{T \vdash \mathrm{enc}(\mathit{secret}, k)} \qquad \dfrac{}{T \vdash k}}{T \vdash \mathrm{dec}(\mathrm{enc}(\mathit{secret}, k), k)}}{T \vdash \mathit{secret}} \quad f \in \Sigma \quad \mathrm{dec}(\mathrm{enc}(x, y), y) = x$$

Rewriting systems

For analyzing equational theories, we (try to) associate to E a finite convergent rewriting system $\mathcal{R}$ such that :

$$u =_E v \quad \text{iff} \quad u\downarrow = v\downarrow$$

Definition (Characterization of the deduction relation)

Let $t_1, \ldots t_n$ and u be terms in normal form.

$$\{t_1, \ldots t_n\} \vdash u \quad \text{iff} \quad \exists C \text{ s.t. } C[t_1, \ldots, t_n] \rightarrow^* u$$

(Also called Cap Intruder problem [Narendran et al])

Some results with equational theories

	Security problem	
	Bounded number of sessions	**Unbounded number of sessions**
Commutative encryption	*co-NP-complete* *[CKRT04]*	**Ping-pong protocols :** *co-NP-complete* *[Turuani04]*
Exclusive Or	*Decidable* *[CS03,CKRT03]*	**One copy - No nonces :** *Decidable* *[CLC03]* **Two-way automata - No nonces :** *Decidable* *[Verma03]*
Abelian Groups	*Decidable* *[Shmatikov04]*	**Two-way automata - No nonces :** *Decidable* *[Verma03]*
Prefix encryption	*co-NP-complete* *[CKRT03]*	
Abelian Groups and Modular Exponentiation	**General case :** *Decidable* *[Shmatikov04]* **Restricted protocols :** *co-NP-complete* *[CKRT03]*	**AC properties of the Modular Exponentiation No nonces :** *Semi-Decision Procedure* *[GLRV04]*

Outline of the talk

Specificity of cryptographic models

- Messages are bitstrings
- Real encryption algorithm
- Real signature algorithm
- General and powerful adversary

$\longrightarrow$ very little abstract model

Encryption nowadays

$\longrightarrow$ Based on algorithmically hard problems.

RSA Function $n = pq$, p et q primes.
e : public exponent

- $x \mapsto x^e \mod n$ easy (cubic)
- $y = x^e \mapsto x \mod n$ difficult
  $x = y^d$ où $d = e^{-1} \mod \phi(n)$

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- Compute $\mathrm{DH}(A, B) = g^{ab}$

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- Given $A = g^a$ and $B = g^b$,
- Compute $\mathrm{DH}(A, B) = g^{ab}$

$\longrightarrow$ Based on hardness of integer factorization.

Estimations for integer factorization

Module (bits)	Operations (in $\log_2$)
512	58
1024	80
2048	111
4096	149
8192	156

$\approx 2^{60}$ years

$\rightarrow$ Lower bound for RSA and Diffie-Hellman.

Setting for cryptographic protocols

Protocol :

- Message exchange program
- using cryptographic primitives

Adversary $\mathcal{A}$: any probabilistic polynomial Turing machine, *i.e.* any probabilistic polynomial program.

- polynomial : captures what is feasible
- probabilistic : the adversary may try to guess some information

Definition of secrecy preservation

→ Several notions of secrecy :

One-Wayness : The probability for an adversary $\mathcal{A}$ to compute the secret s against a protocol $\mathcal{P}$ is negligible (smaller than any inverse of polynomial).

$$\forall p \text{ polynomial } \exists \eta_0 \, \forall \eta \geq \eta_0 \quad \Pr^{\eta}_{m,r}[\mathcal{A}(\mathcal{P}_K) = s] \leq \frac{1}{p(\eta)}$$

η : security parameter = key length

Not strong enough !

- The adversary may be able to compute half of the secret message.
- There is no guarantee in case that some partial information on the secret is known.

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→ Introduction of a notion of indistinguishability.

Indistinguishability

The secrecy of s is defined through the following game :

- Two values n_0 and n_1 are randomly generated instead of s ;
- The adversary interacts with the protocol where s is replaced by n_b, $b \in \{0, 1\}$;
- We give the pair (n_0, n_1) to the adversary ;
- The adversary gives b',

The data s is secret if $\Pr[b = b'] - \frac{1}{2}$ is a negligible function.

A typical cryptographic proof

1. Assume that some algorithmic problem P is difficult (E.g. RSA or integer factorization or Discrete Log or CDH, DDH, ...)
2. Suppose that a (polynomial probabilistic) adversary $\mathcal{A}$ breaks the protocol security with non negligible probability

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4. Conclude that the protocol is secure provided P is difficult.

Formal and Cryptographic approaches

	Formal approach	Cryptographic approach
Messages	terms	bitstrings
Encryption	idealized	algorithm
Adversary	idealized	any polynomial algorithm
Secrecy property	reachability-based property	indistinguishability
Guarantees	unclear	strong
Protocol	may be complex	usually simpler

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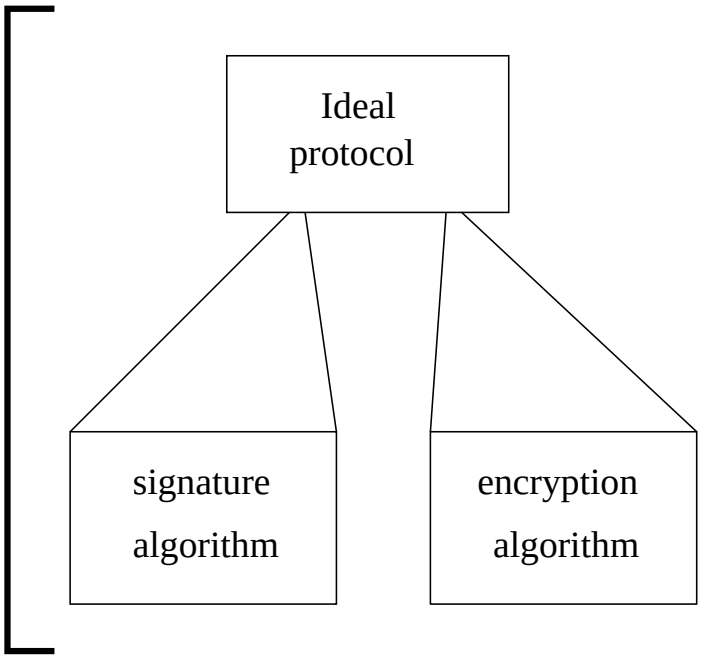
Guarantees	unclear	strong
Protocol	may be complex	usually simpler
Proof	automatic	by hand, tedious and error-prone

Link between the two approaches ?

Composition of the two approaches

Automatic cryptographically sound proofs

Implemented protocol



Formal approach: verification of idealized protocols

Cryptographers: verification of the cryptographic primitives

Example : correspondence of secrecy properties

Theorem

For protocols with only public key encryption, signatures and nonces,
Whenever a protocol is proved to ensure secrecy in formal models then it ensures cryptographic indistinguishability in the computational models.

Hypotheses on the Implementation

- encryption : IND-CCA2 (e.g. the OAEP-RSA scheme)
  $\longrightarrow$ the adversary cannot distinguish between $\{n_0\}_k$ and $\{n_1\}_k$ even if he has access to n_0 and n_1 and to encryption and decryption oracles.
- signature : existentially unforgeable under chosen-message attack *i.e.* one can not produce a valid pair (m, σ)
- parsing :
  - each bit-string has a label which indicates his type (identity, nonce, key, signature, ...)
  - one can retrieve the (public) encryption key from an encrypted message.
  - one can retrieve the signed message from the signature

Proof technique

Lemma (Mapping lemma)

Each execution trace of a concrete adversary is captured by a symbolic execution trace of an ideal adversary, except with negligible probability

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Example : If a computational (concrete) adversary $\mathcal{A}$ is able to compute $\{n_a\}_{K_a}$ out of $\{< A, n_a >\}_{K_a}$,
Then we can build an adversary $\mathcal{A}'$ that breaks the encryption $\{\ \}_{K_a}$.

Conclusion

Formal methods form a powerful approach for analyzing security protocols

- Makes use of classical techniques in formal methods : term algebra, equational theories, clauses and resolution techniques, tree automata, etc.
  $\Rightarrow$ Many decision procedures
- Several automatic tools
  - For successfully detecting attacks on protocols (e.g. Casper, Avispa)
  - For proving security for an arbitrary number of sessions (e.g. ProVerif)
- Provides cryptographic guarantees under classical assumptions on the implementation of the primitives

Some current directions of research

- Enriching the symbolic model
  - Considering more equational theories (e.g. theories for e-voting protocols)
  - Adding more complex structures for data (list, XML, ...)
  - Considering recursive protocols (e.g. group protocol) where the number of message exchanges in a session is not fixed
  - Proving more complex security properties like equivalence-based properties (e.g. for anonymity or e-voting protocols)
- With cryptographic guarantees
  - Combining formal and cryptographic models for more complex primitives and security properties.
  - How far can we go ?
  - Is it possible to consider weaker cryptographic primitives ?